Int. J. Heat Mass Transfer. Vol. 34, No. 11, pp. 2883–2889, 1991
Printed in Great Britain

0017-9310/91 $3.00+0.00
Pergamon Press plc

Conjugate heat and mass transfer in continuous processes of convective drying

A. A. DOLINSKIY, A. SH. DORFMAN and B. V. DAVYDENKO

Institute of Engineering Thermophysics, Academy of Sciences of the Ukr.SSR, Kiev, 252057, U.S.S.R.

(*Received* 26 *October* 1989)

Abstract—Based on the previously developed mathematical model, heat and mass transfer in the initial period of drying and moistening of a continually moving material is studied in a conjugate statement of the problem. This approach allows one not only to take account of the effect of the external conditions on the processes in the material but also of the reverse influence which is exerted by these processes on the heat and mass transfer and which, in principle, cannot be allowed for by traditional calculational techniques based on the boundary conditions of the third kind.

A CONJUGATE problem of heat and mass exchange between a heat-transfer agent and a continuous material pulled through it was formulated and solved in ref. [1] using, in lieu of the generally employed third-kind boundary conditions, the conditions of the fourth-kind which allow one to take into account the interaction effect of the heat and mass exchange processes in the material being pulled and in the heat-transfer agent. In ref. [1] the material was taken to be thin, and the thickness averaged equations of heat and mass transfer were considered for it.

The present work is concerned with the same conjugate problem for a material of arbitrary thickness, pulled through a heat-transfer agent. Accordingly, a system of differential (non-thickness-averaged) equations of heat and mass conductivity is solved for the body. The model considered is represented schematically in Fig. 1. The material of thickness Δ is pulled at the speed V_x. The quantities known are: the thermophysical properties of the material and heat-transfer agent, the temperature T_∞, the pressure p_∞, the density ρ_∞ of the heat-transfer agent far from the body, and the relative concentration $\rho_{10,\infty}$ of vapour in it. The prescribed quantities are: the temperature $T(0)$ and the moisture content $u(0)$ of the material in the initial section $(x = 0)$. The solution of the conjugate problem provides all necessary characteristics of the process including the heat and mass transfer coefficients. In such a formulation the problem is described by a system of equations [1] consisting of transfer equations for the moist material:

$$C_{\text{eff}}\rho_3 V_x \frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(\lambda_{\text{eff}}\frac{\partial T}{\partial y}\right) + \rho_3 \varepsilon r \frac{\partial u}{\partial x} \quad (1)$$

$$\rho_3 V_x \frac{\partial u}{\partial x} = \frac{\partial}{\partial y}\left[\partial_3 a_{\text{m}}\left(\frac{\partial u}{\partial y} + \delta\frac{\partial T}{\partial y}\right)\right] \quad (2)$$

under initial conditions

$$x = 0 \quad T = T(0), \quad u = u(0) \quad (3)$$

conditions of symmetry along the material axis

$$y = 0 \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial u}{\partial y} = 0 \quad (4)$$

and conditions of conjugation on the material surface

$$\frac{\rho_\infty D}{1-\rho_{10,\infty}}\left.\frac{\partial \rho_{10}}{\partial y}\right|_{\text{w}} = \rho_3 a_{\text{m}}\left(\left.\frac{\partial u}{\partial y}\right|_{\text{w}} + \delta\left.\frac{\partial T}{\partial y}\right|_{\text{w}}^{-}\right) \quad (5)$$

$$\lambda\left.\frac{\partial T}{\partial y}\right|_{\text{w}}^{+} - \lambda_{\text{eff}}\left.\frac{\partial T}{\partial y}\right|_{\text{w}}^{-} = -(1-\varepsilon_{\text{w}}) r a_{\text{m}}\rho_3\left(\left.\frac{\partial u}{\partial y}\right|_{\text{w}} + \delta\left.\frac{\partial T}{\partial y}\right|_{\text{w}}^{-}\right) \quad (6)$$

$$\frac{1}{\phi(T_{\text{w}}, u_{\text{w}})} = \frac{p_{\text{s}}(T_{\text{w}})}{p_\infty}\left[1 + \frac{R_0}{R_1}\left(\frac{1}{\rho_{10,\text{w}}} - 1\right)\right]. \quad (7)$$

These conditions were obtained in ref. [1]. The first condition expressed the quality of mass fluxes calculated at the surface on the heat-transfer agent side (+) and on the material side (−), the second condition gives the heat balance on the surface and the third condition presents the equation of desorption isotherms and relates the temperature T_{w} and the moisture content u_{w} at the material surface to the relative concentration of vapours $\rho_{10,\text{w}}$ at the surface.

Relations (1)–(7) are augmented with the expressions for the heat and mass fluxes

$$q_{\text{w}} = \alpha_*\left(T_{\text{w}} - T_\infty + g_1 x\frac{\text{d}T_{\text{w}}}{\text{d}x} + g_2 x^2\frac{\text{d}^2 T_{\text{w}}}{\text{d}x^2} + \cdots\right) \quad (8)$$

$$j_{\text{w}} = \beta_*\left(\rho_{10,\text{w}} - \rho_{10,\infty} + h_1 x\frac{\text{d}\rho_{10,\text{w}}}{\text{d}x} + h_2 x^2\frac{\text{d}^2\rho_{10,\text{w}}}{\text{d}x^2} + \cdots\right). \quad (9)$$

NOMENCLATURE

a	thermal diffusivity [$m^2 s^{-1}$]
a_m	moisture diffusivity [$m^2 s^{-1}$]
c_1, c_2	exponents
C	heat capacity [$J kg^{-1} K^{-1}$]
D	vapour diffusion coefficient [$m^2 s^{-1}$]
d_1, d_2	exponents
f_w	injection parameter
Fo	Fourier number
j	mass flux [$kg m^{-2} s^{-1}$]
Le	Lewis number
p	pressure [$N m^{-2}$]
Pr	Prandtl number
q	heat flux [$W m^{-2}$]
r	heat of evaporation [$J kg^{-1}$]
R	gas constant [$J kg^{-1} K^{-1}$]
Sc	Schmidt number
T	temperature [K]
u	moisture content [$kg kg^{-1}$]
V_x	speed [$m s^{-1}$]
x	longitudinal coordinate [m]
y	vertical coordinate [m].

Greek symbols

α	heat transfer coefficient [$W m^{-2} K^{-1}$]
β	mass transfer coefficient [$kg m^{-2} s^{-1}$]
δ	temperature-gradient coefficient [K^{-1}]
Δ	material thickness [m]
ε	phase change coefficient
λ	thermal conductivity [$W m^{-1} K^{-1}$]
ν	kinematic viscosity [$m^2 s^{-1}$]
ρ	density [$kg m^{-3}$]
ρ_{10}	relative concentration of vapour, $\rho_1/(\rho_1+\rho_0)$
τ	time [s]
ϕ	relative humidity
χ_t	coefficient of non-isothermicity
χ_p	coefficient of partial non-isobaricity.

Subscripts

0	air
1	vapour
2	liquid
3	dry material
eff	effective, moist material
m.s.	maximum sorptive
p	constant pressure
s	vapour saturated
w	wall
∞	far from the body
*	constant head.

In contrast to the usual third-kind boundary conditions, the above expressions were obtained by solving the thermal and diffusional boundary-layer equations [1, 2] and enable one to incorporate the effect of the temperature and concentration distribution over the surface on the heat and mass fluxes. With the temperature and concentration on the surface being invariant, these expressions are converted to the third-kind boundary conditions. Equations (8) and (9) can be replaced by their integral analogues [1, 2]

$$q_w = \alpha_* \left\{ T_w(0) - T_\infty + \int_0^x \left[1 - \left(\frac{\xi}{x}\right)^{c_1} \right]^{-c_2} \frac{dT_w}{d\xi} d\xi \right\} \quad (10)$$

$$j_w = \beta_* \left\{ \rho_{10,w}(0) - \rho_{10,\infty} + \int_0^x \left[1 - \left(\frac{\xi}{x}\right)^{d_1} \right]^{-d_2} \frac{d\rho_{10,w}}{d\xi} d\xi \right\}. \quad (11)$$

At the constant temperature and concentration on the surface the heat (α_*) and mass (β_*) transfer coefficients are defined as

$$\alpha_* = g_0(Pr, f_w)\lambda\sqrt{(V_x/\nu x)}; \quad \beta_* = (\alpha_*/C_p)Le^{1/2}. \quad (12)$$

In the above expressions λ_{eff}, a_m and C_{eff} are the coefficients of thermal conductivity and diffusion, and heat capacity of the moist material, respectively; $p_s(T_w)$ the pressure of saturated vapour at the surface; $\phi = \rho_{1,w}/\rho_s(T_w)$, $\rho_{1,w}$ the density of vapour at the surface; Le the Lewis number which is practically 1 for gases; g_0, g_k, h_k, c and d the tabulated coefficients and exponents depending either on the injection parameter f_w or on the Prandtl number Pr in equations (8) and (10), or on the Schmidt number Sc in equations (9) and (11).

In ref. [1] equations (1) and (2) were averaged over the thickness, and the above-given system of equations was used to predict the convective drying of a thin material. The problem was reduced to numerical integration of two first-order ordinary integro-differential equations. As an example, the drying of filter paper in the second period, when the moisture content of the material is smaller than its maximum sorptive value, was considered.

In the present work attention is focused on heat and mass exchange between the material and the heat-transfer agent in the initial period when moisture content of the material varies from initial to maximum sorptive. Here, unlike the case in ref. [1] the material is not assumed to be thin and equations (1) and (2) of transfer in the body are not averaged. Heat is supplied to the material only by convection, the flow regime is laminar and the thermal physical properties of the moist air are calculated additively. Calculations show that, with the evaporation rates usual for the convective drying, the influence of the injection

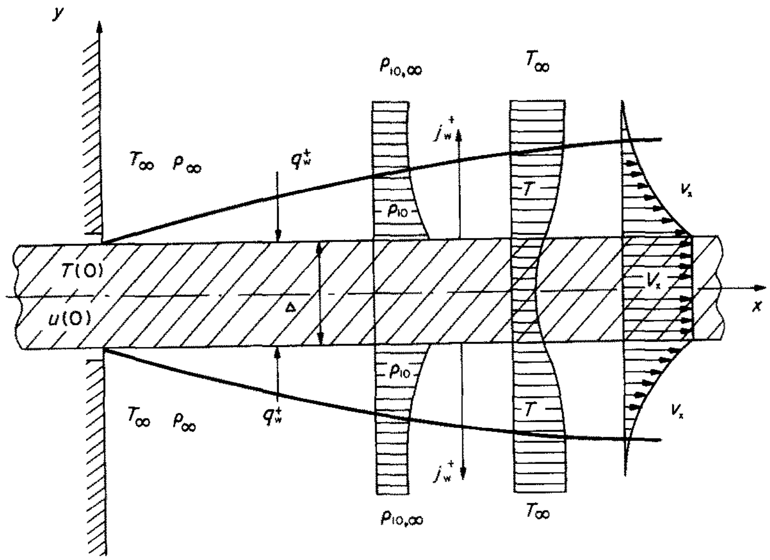

FIG. 1. Model of the process.

parameter can be disregarded [3]. In this case, for the laminar flow without injection and with $Pr = Sc = 0.7$, the coefficients in equations (8)–(12) have the following values [4]: $g_0 = 0.351$; $g_1 = h_1 = 1.35$; $g_2 = h_2 = -0.18$; $g_3 = h_3 = 0.03$; $c_1 = d_1 = 1.2$; $c_2 = d_2 = 0.57$.

The proposed calculation technique also holds for more complicated cases, for example, for the turbulent flow regime, conductive–convection heat input, auxiliary heat supply by radiation, as well as in some other cases. With variation of the flow regime, equations (8)–(12) should incorporate the corresponding values of the coefficients g_k and h_k and exponents c and d. Their values should be established in the manner usually adopted for determining the heat transfer coefficients either by solving the appropriate system of boundary-layer equations, or experimentally [2]. In the conductive or radiative heat input case, the condition of symmetry on the axis equations (4), should be substituted by the corresponding condition of heat supply to the material. The remaining relations are retained.

For the initial period under consideration, when the moisture content of the material exceeds the maximum sorptive moisture content $u_{\mathrm{m.s.}}$, the partial pressure of the vapours above the surface equals the saturation pressure. In view of this, $\phi = 1$. Besides, when $u > u_{\mathrm{m.s.}}$, the phase change coefficient $\varepsilon = 0$. In this case, equations (1), (6) and (7) simplify.

The dimensionless variables are defined as

$$Fo = \frac{x\lambda_3}{c_3\rho_3 V_x \Delta^2} = \frac{\tau a_3}{\Delta^2}; \quad \eta = \frac{y}{\Delta}$$

$$\theta = \frac{T - T_\infty}{T(0) - T_\infty}; \quad U = \frac{u - u_{\mathrm{m.s.}}}{u(0) - u_{\mathrm{m.s.}}} \tag{13}$$

where τ is the time, a the thermal diffusivity, and Fo the Fourier number. In terms of these variables equations (1) and (2) take on the form

$$\frac{C_{\mathrm{eff}}}{C_3}\frac{\partial \theta}{\partial Fo} = \frac{\partial}{\partial \eta}\left(\frac{\lambda_{\mathrm{eff}}}{\lambda_3}\frac{\partial \theta}{\partial \eta}\right) \tag{14}$$

$$\frac{\partial U}{\partial Fo} = \frac{\partial}{\partial \eta}\left[\frac{a_{\mathrm{m}}}{a_3}\left(\frac{\partial U}{\partial \eta} + \delta \frac{T(0) - T_\infty}{u(0) - u_{\mathrm{m.s.}}}\frac{\partial \theta}{\partial \eta}\right)\right]. \tag{15}$$

Initial conditions (3) in the section $x = 0$ and boundary conditions (4) on the symmetry of equations (13) are formulated as

$$\theta(0) = 1; \quad U(0) = 1; \quad \partial\theta/\partial\eta = 0; \quad \partial U/\partial \eta = 0. \tag{16}$$

Before turning to variables (13) in conjugation conditions (5)–(7), it is convenient first to carry out their transformation. The expression on the right-hand side of equation (6) will be replaced by the term on the left-hand side of equation (5). Taking into account the fact that for the period considered, when $u > u_{\mathrm{m.s.}}$, the phase change coefficient $\varepsilon = 0$, the derivative $\partial T/\partial y|_{\mathrm{w}}^{-}$ will be found from equations (6) and the derivative $\partial u/\partial y|_{\mathrm{w}}$ will be obtained from equation (5). Thereafter the expressions $\lambda \partial T/\partial y|_{\mathrm{w}}^{+}$ and $\rho_\infty D \partial \rho_{10}/\partial y|_{\mathrm{w}}$, that determine the convective heat and diffusion mass fluxes approaching the surface on the side of the heat-transfer agent, will be substituted by relations (10) and (11) using equations (12) and (13). Moreover, taking into account that for $u > u_{\mathrm{m.s.}}$ the vapour density is equal to the saturation density, the derivative $\mathrm{d}\rho_{10,\mathrm{w}}/\mathrm{d}\xi$ will be presented in the form of the product $(\mathrm{d}\rho_{10,\mathrm{w}}/\mathrm{d}T_{\mathrm{w}})(\mathrm{d}T_{\mathrm{w}}/\mathrm{d}\xi)$. These transformations yield the following relations:

$$-\frac{\partial\theta}{\partial\eta}\bigg|_{w} = \frac{g_0(Pr)\lambda_3}{\lambda_{eff}}\sqrt{\left(\frac{b}{Pr\,Fo}\right)}$$
$$\times\left\{1+\int_0^{Fo}\left[1-\left(\frac{Fo'}{Fo}\right)^{c_1}\right]^{-c_2}\frac{d\theta_w}{dFo}\,dFo'\right.$$
$$+\frac{r}{(1-\rho_{10,w})[C_{p_1}\rho_{10,w}+C_{p_0}(1-\rho_{10,w})]}$$
$$\times\left\{\frac{\rho_{10,w}(0)-\rho_{10,\infty}}{T(0)-T_\infty}+\int_0^{Fo}\left[1-\left(\frac{Fo'}{Fo}\right)^{d_1}\right]^{-d_2}\right.$$
$$\left.\left.\times\frac{d\rho_{10,w}}{dT_w}\frac{d\theta_w}{dFo'}\,dFo'\right\}\right\} \quad (17)$$

$$-\frac{\partial U}{\partial\eta}\bigg|_{w} = \frac{g_0(Pr)C_3}{(a_m/a_3)(1-\rho_{10,w})[C_{p_1}\rho_{10,w}+C_{p_0}(1-\rho_{10,w})]}$$
$$\times\sqrt{\left(\frac{b}{Pr\,Fo}\right)}\left\{\frac{\rho_{10,w}(0)-\rho_{10,\infty}}{u(0)-u_{m.s.}}+\frac{T(0)-T_\infty}{u(0)-u_{m.s.}}\right.$$
$$\left.\times\int_0^{Fo}\left[1-\left(\frac{Fo'}{Fo}\right)^{d_1}\right]^{-d_2}\frac{d\rho_{10,w}}{dT_w}\frac{d\theta_w}{dFo'}\,dFo'\right\}$$
$$+\delta\frac{T(0)-T_\infty}{u(0)-u_{m.s.}}\frac{\partial\theta}{\partial\eta}\bigg|_{w} \quad (18)$$

where $b = C\rho\lambda/(C\rho\kappa)_3$ is the parameter characterizing the relationship between the thermophysical properties of the heat-transfer agent and material, Fo' the integration variable replacing the variable ξ. The conjugation condition, equation (7), for the period being considered, for which $\phi = 1$, is employed to determine the relative vapour concentration

$$\rho_{10,w} = \frac{1}{1+\frac{R_1}{R_0}\left[\frac{p_\infty}{p_s(T_w)}-1\right]}. \quad (19)$$

Analysing the system of equations as well as the initial, boundary and conjugation conditions, stated in dimensionless variables (13), enables the following conclusions to be drawn which are valid for the first period of drying:

(a) The considered conjugate problem for the fixed material and heat-transfer agent is governed by four parameters: $\rho_{10,\infty}$, $T(0)$, $u(0)$ and T_∞.

(b) When the dependence of the material properties on temperature and moisture content is neglected and calculations are confined to the mean values of thermophysical coefficients, the number of parameters that govern the problem at fixed material and heat-transfer agent is reduced to two:

$$\frac{T(0)-T_\infty}{u(0)-u_{m.s.}},\quad \frac{\rho_{10,w}-\rho_{10,\infty}}{u(0)-u_{m.s.}}.$$

(c) The body thickness is incorporated only in the Fourier number. Therefore, the time needed for the material to reach a definite state characterized by certain values of temperature and moisture content and the drying duration are proportional to the material thickness squared.

(d) The velocity at which the material is pulled through the heat-transfer agent does not affect the drying rate and time.

It only determines the distance from a die over which the material reaches a certain state. The explanation is that the heat and mass fluxes in the laminar flow regime are inversely proportional to the ratio x/V_x, i.e. to the time of the material pulling as follows from equations (8), (9) and (12). It can be readily demonstrated that in the turbulent flow regime the drying duration will depend on the pulling velocity, since the coefficients α_* and β_*, defined by the formulae such as equations (12), will be proportional to $V_x^{0.6}$.

Equations (14) and (15) under initial, boundary, equations (16), and conjugation, equations (17)–(19), conditions were solved numerically by the difference technique using the tridiagonal matrix algorithm and the implicit difference scheme. First, equation (14) was solved with condition (17) and thereafter equation (15) with condition (18) which contained the already known derivative $\partial\theta/\partial\eta|_w$ calculated when solving equation (14). To calculate the equation coefficients that depend on the functions sought, iterations were carried out.

Calculations were performed for the following conditions: $T_\infty = 90°C$, $\rho_{10,\infty} = 0.125$ and $u(0) = 0.25$. Two cases were considered: with the initial temperature of the material equal to $T(0) = 70$ and $50°C$. The first of them is higher, and the second is lower, than the dew-point temperature corresponding to the assigned relative vapour concentration in the heat-transfer agent $\rho_{10,\infty} = 0.125$. Therefore, in the first case drying proceeds from the very beginning whereas in the second case the material is first moistened, and drying begins only after some time interval.

The calculations were made for the thermophysical properties characteristic of the paper-type material; $\rho_3 = 800$ kg m^{-3}, $C_3 = 1500$ J kg^{-1} K^{-1}, $C_{eff} = C_3 + C_{H_2O}u$; the maximum sorptive moisture content at the dew-point temperature $u_{m.s.} = 0.19$ [5], the thermal conductivity of the moist material was taken to be constant $\lambda_{eff} = 0.4$ W m^{-1} K^{-1}. As evident from the equations and conjugation conditions, the Fourier number and the governing parameters contain the quantity λ_{eff} instead of λ_3. For the parameter which determines the relationship between the thermophysical properties and the dimensionless coefficient of moisture diffusion (at the adopted value of a_m) it is found that $b = C\rho\lambda/(C\rho)_3\lambda_{eff} = 6\times10^{-5}$ and $a_m(C\rho)_3/\lambda_{eff} = 0.125$. In accordance with the foregoing, the thickness and the pulling velocity of the material were not specified.

Predictions show that, in the conditions under consideration, the temperature and moisture content vary little across the material and their values on the surface do not actually differ from those mean integrals over the thickness. Figures 2 and 3 illustrate time depen-

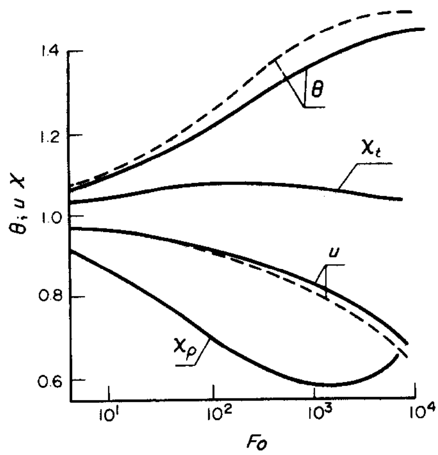

FIG. 2. Variations in time of the dimensionless parameters during the drying process. Dashed lines, calculation for the third-kind boundary conditions.

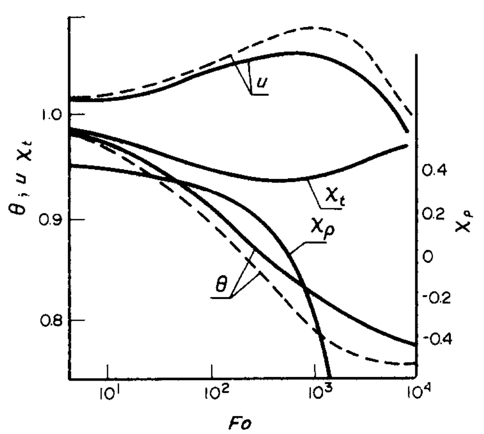

FIG. 3. Variations in time of the dimensionless parameters during the moistening process. Dashed lines, calculation for the third-kind boundary conditions

dence of the dimensionless temperature and moisture content on the material surface. Figure 2 pertains to the case with the material temperature higher than the dew point and Fig. 3 to the case with this temperature lower than that of the dew point. Solid lines show the results of the solution of the conjugate problem when the fourth-kind boundary conditions are employed. Dashed lines demonstrate the results calculated for the third-kind boundary conditions using the heat and mass transfer coefficients α_* and β_* calculated from equations (12), which are valid at constant temperature and concentration heads along the plate.

In solving the conjugate problem, the heat and mass fluxes are determined by equations (8) and (9) or (10) and (11). This allows one to calculate the real heat and mass transfer coefficients with regard for the distribution of the temperature and concentration heads established on the surface due to the interaction between the material and heat-transfer agent. To evaluate the conjugation effect, the coefficients of non-isothermicity and partial isobaricity [4]

$$\chi_t = \frac{\alpha}{\alpha_*}, \quad \chi_p = \frac{\beta}{\beta_*}$$

are introduced which determine the extent to which the heat and mass transfer coefficients, obtained for the real distributions of the temperature and concentration heads, differ from the usually employed coefficients α_* and β_* obtained under the assumption of constant heads. The values of the coefficients χ_t and χ_p for the cases under consideration are also given in Figs. 2 and 3.

The analysis of the calculated results allows one to draw the following conclusions:

(a) The real rate of the heat and mass transfer processes predicted with allowance for the interaction between the material and the heat-transfer agent, is lower than that resulting from the calculation conducted for the third-kind boundary conditions in which the conjugate character of the problem was disregarded. This is attributable to the fact that in the case of drying, the solid curve of moisture content is located higher (Fig. 2), and in the case of moistening, lower (Fig. 3), than the corresponding dashed curves. Accordingly, the solid curves of temperature are situated lower for drying (Fig. 2) and higher for moistening (Fig. 3).

(b) The reason for such a relationship between the rates of the heat and mass transfer processes is traced to the difference between the heat and mass transfer coefficients, the relationship between which, as is known, is determined by the character of variation in the temperature and concentration heads [2, 6]. When the head grows, either in the direction of the heat-transfer agent flow or in time, the heat and mass transfer coefficients appear to be higher, where in the reverse case they turn out to be lower than the corresponding coefficients obtainable for the constant head. In the considered drying and moistening process, the concentration heads diminish.The temperature head increases in the drying process and decreases during the moistening process. In conformity with this, the mass transfer coefficients in both the cases are smaller than $\beta_*(\chi_p < 1)$. The heat transfer coefficient is smaller than α_* for moistening ($\chi_t < 1$) and larger than α_* for drying ($\chi_t > 1$). It is precisely this factor that stipulates the decrease in the heat and mass transfer rate as compared with that acquired by the traditional calculation for the third-kind boundary conditions taking no account of the conjugate character of the problem. Here the heat transfer rate decreases also in drying even though $\alpha > \alpha_*$. This is associated with a much more appreciable decrease in the mass transfer coefficient than the increase of the heat transfer coefficient in the drying process considered (Fig. 2).

Since the processes of convective drying and moistening proceed under the conditions of the falling concentration head, the established decrease in the rate will take place in any process of this type. Naturally, the quantitative results and the degree of this decrease will be determined by the specific conditions but qualitative results will be the same.

(c) The analogy between the heat and mass transfer coefficients, frequently employed in predictions, is not observed. This is due to the fact that, for maintaining the analogy, the coincidence is required not only of the differential equations describing heat and mass transfer but also of the appropriate boundary conditions determined by the distribution of the temperature and concentration heads along the surface or in time. It is seen that these distributions substantially differ. Correspondingly, the distributions of the heat and mass transfer coefficients also considerably differ, quantitatively and qualitatively. While the heat transfer coefficients differ little from the isothermal coefficients α_*, so that the non-isothermicity coefficients χ_t are close to unity, the mass transfer coefficients differ drastically from the isobaric ones β_*, especially in the process of moistening (Fig. 3). Here the coefficient of partial non-isobaricity χ_p reaches 0.6 in one case (Fig. 2), while in the other it even reduces to zero and becomes negative. Thus, in the processes considered, the inclusion of the interaction between the material and the heat-transfer agent has little effect on the heat transfer coefficients and noticeably reduces the mass transfer coefficients.

(d) In the course of moistening there occurs the mass flow inversion—the phenomenon similar to the well-studied phenomenon of the heat flux inversion [2, 6]. It follows from the law of proportionality between the flux and the head that the moistening process should persist until the material temperature becomes equal to the dew-point temperature and the partial vapour pressures above the surface and far from it become identical. In actual fact, the mass flux reduces to zero earlier than the concentration head. This is illustrated in Fig. 4 which presents variations in the mass flux j_w and concentration head $\Delta\rho_{10}$ obtained in the conjugate and non-conjugate problems. It is seen that the mass flux reduces to zero much earlier than the concentration head does. As is to be expected, in the non-conjugate problem (dashed lines) both the points coincide. At the point where the mass flux reduces to zero the concentration head is finite and, therefore, the mass transfer coefficient and, consequently, the non-isobaricity coefficient χ_p become zero. After this point, the mass flux changes its sign, and the drying process begins even though the direction of the concentration head remains the same ($\rho_{10,w} < \rho_{10,\infty}$). Therefore, the mass transfer coefficient and, correspondingly, the non-isobaricity coefficient χ_p are negative over this section. Such a pattern remains up to the point at which the concentration head vanishes. Since the mass flux is finite, the mass transfer and non-isobaricity coefficients at this point tend to infinity and virtually lose meaning.

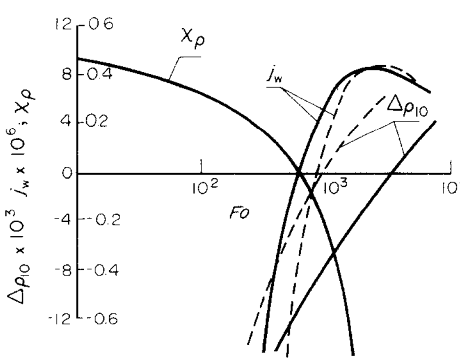

FIG. 4. Variations of the mass flux and concentration head with inversion. Dashed lines, calculation for the third-kind boundary conditions.

Physically, the inversion phenomenon is explained by the inertia properties of the heat-transfer agent flow, due to which the change in concentration near the wall is manifested much earlier in its immediate vicinity than far from the wall. This results in the fact that, when the concentrations of the heat-transfer agent on the wall and in its immediate vicinity become the same and the mass flux reduces to zero, the concentration far from the wall does not manage to become equal to that on the wall and, hence, the concentration head does not vanish. The inversion in application to the heat flux is studied in more detail in refs. [2, 7].

The inversion-involving processes cannot be investigated and calculated by traditional methods, since the third-kind boundary conditions essentially contradict the inversion phenomenon. In such cases the problem should be regarded as a conjugate one.

CONCLUSION

Based on the previously developed mathematical model, heat and mass exchange are studied in the initial period of drying and moistening of a continuously moving material in the conjugate statement of the problem. This statement allows one to take into account not only the effect of the external conditions on the processes taking place in the material, but also the inverse influence of these processes on the heat and mass transfer rate, which in principle cannot be considered by the traditional calculational techniques based on the third-kind boundary conditions.

It is found that the solution of the conjugate problem yields decreased rates of drying and moistening as against those obtained by traditional methods. It is shown that the frequently used analogy between the heat and mass transfer coefficients is not observed and that the actual heat and mass transfer coefficients differ considerably both from each other and from the values obtained by ordinary similarity equations. Under certain conditions, in the process of moistening the phenomenon of the mass flux inversion originates which basically cannot be described by the generally employed laws of direct proportionality between the fluxes and heads. Account is also given of some other features of the conjugate formulation of the problems of combined heat and mass exchange.

The results presented are obtained under certain assumptions, for the laminar flow regime. It can be demonstrated, however, that they are also qualitatively valid for the turbulent flow regime and for the real drying processes in which the convective heat input is essential. This is due to the fact that, during the drying processes, the concentration head diminishes either along the material length or in time. The transfer coefficients for decreasing heads are always noticeably smaller than the corresponding coefficients calculated under the assumption of constant heads. Therefore, the actual drying process, just as in the problems considered, will proceed more slowly than it follows from the conventional traditional calculations. Of course, in each specific case the qualitative results should be found by calculation and can differ from those given in the present study.

The solution of conjugate problems enables one to more profoundly understand the physical essence of the drying and moistening processes, to appreciably improve the accuracy of calculations, and to establish the limits of applicability of the approximate traditional calculational technique. The studies of the problems of conjugate heat and mass exchange are in their early stages. They should be continued by expanding the range of the problems considered and by working out the methods and programs to be widely employed in the calculation practice.

REFERENCES

1. O. A. Grechannyi, A. A. Dolinskiy and A. Sh. Dorfman, Conjugate heat and mass transfer in continuous processes of the convective drying of thin bodies, *Prom. Teplotekhn.* **9**(4), 27–37 (1987).
2. A. Sh. Dorfman, *Heat Transfer in Flow Over Non-isothermal Bodies.* Izd. Mashinostroyeniye, Moscow (1982).
3. A. V. Luikov, *Drying Theory.* Izd. Energiya, Moscow (1968).
4. A. Sh. Dorfman and V. G. Novikov, Heat exchange between a continuously moving plate and the surrounding medium, *Teplofiz. Vysok. Temp.* **18**(6), 1203–1210 (1980).
5. L. M. Nikitina, *Tables of the Mass Transfer Coefficients of Moist Materials.* Izd. Nauka i Tekhnika, Minsk (1964).
6. O. A. Grechannyi, A. A. Dolinskiy and A. Sh. Dorfman, Effect of the nonuniformity of the distributions of temperature and concentration heads on heat and mass transfer from a continuously moving porous plate, *Prom. Teplotekhn.* **9**(2), 19–29 (1987).
7. E. R. G. Eckert and R. M. Drake, *Heat and Mass Transfer.* McGraw-Hill, New York (1959) (Russian transl. (1961)).

TRANSFERTS CONJUGUES DE CHALEUR ET DE MASSE DANS DES MECANISMES CONTINUS DE SECHAGE CONVECTIF

Résumé—On étudie du point de vue du problème conjugué le transfert de chaleur et de masse dans la période initiale du séchage et de l'humidification d'un matériau se déplaçant continûment, à partir d'un modèle mathématique développé antérieurement. Cette approche permet non seulement de tenir compte de l'effet des conditions externes sur les mécanismes à l'intérieur du matériau mais aussi de l'influence inverse qui est exercée par ces mécanismes sur le transfert de chaleur et de masse, ce qui ne peut en principe pas être traité par les techniques traditionnelles de calcul basées sur les conditions aux limites de troisième espèce.

KONJUGIERTER WÄRME- UND STOFFTRANSPORT BEI KONTINUIERLICH ABLAUFENDEN KONVEKTIVEN TROCKNUNGSPROZESSEN

Zusammenfassung—Aufbauend auf einem früher entwickelten mathematischen Modell wird der Wärme- und Stofftransport in der Anfangsphase der Trocknung und Befeuchtung eines sich kontinuierlich bewegenden Materials untersucht, wobei die konjugierten Koppelungen bei dem Problem berücksichtigt werden. Dieses Vorgehen erlaubt nicht nur eine Berücksichtigung des Einflusses der äußeren Bedingungen auf die Vorgänge im Material, sondern auch Rüchwirkungen, welche von diesen Prozessen auf den Wärme- und Stofftransport ausgehen. Derartige Rückwirkungen können bei herkömmlichen Berechnungsverfahren aufgrund von Randbedingungen der dritten Art grundsätzlich nicht berücksichtigt werden.

СОПРЯЖЕННЫЙ ТЕПЛОМАССООБМЕН В НЕПРЕВЫНЫХ ПРОЦЕССАХ КОНВЕКТИВНОЙ СУШКИ

Аннотация—На основе ранее разработанной математической модели исследован тепломассообмен в начальном периоде сушки и увлажнения непрерывно движущегося материала при сопряженной постановке задачи. Такой подход позволяет учесть не только влияние внешних условий на процессы в материале, но и обратное влияние процессов в материале на интенсивность тепло- и массоотдачи, которое в традиционных методах расчета, основанных на граничных условиях третьего рода, принципиально не может быть учтено.